GL Assessment

Assessment in Focus

Maths competencies and progress in learning around the world: PISA and Progress Test in Maths

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Maths competencies and progress in learning around the world: PISA and Progress Test in Maths

Introduction

This document compares and contrasts the Mathematics Framework in the Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment (PISA) and GL Assessment's *Progress Test in Maths* (*PTM*) 5-14.[1]

Mathematical concepts and skills are the same around the world. Both OECD and GL Assessment have created assessments of mathematical skills and knowledge that build on this global foundation of mathematical competencies. When developing the PISA Mathematics Framework and the 11 tests that make up the *Progress Test in Maths* series, the latest research on mathematics teaching and learning was included. Reference to local curricula in different countries and regions was made and thus incorporated into the assessment frameworks. With this shared background, it is not surprising that the two assessments have significant overlap in their objectives and content. They both measure similar mathematical process categories and content domains.

However, the purpose of each assessment is different. The PISA framework has been developed to measure education systems with a triennial sample of 15-year-olds. The *PTM* series assesses the skills and knowledge of 5- to 14-year-old individuals in an age-appropriate way.

As the content of the assessments derives from similar sources, it can be assumed that, on average, a student who consistently performs well in *PTM* year-on-year, and progresses as expected for their age, would also perform well in PISA.

To demonstrate improvement of an education system, focus must be on the individual students, and in supporting them to do their best. *PTM* provides detailed profiles on mathematical learning of groups and students, and data that are relevant to each individual child's teaching and learning.

[1] This document draws on information available on OECD's PISA website and other publicly available PISA documentation. A full list of sources is provided at the end of this document.

What is the PISA Mathematics Framework?

PISA is a triennial assessment of education systems administered to a sample of 15-year-olds in the participating countries.

> "The aim of PISA with regard to mathematical literacy is to develop indicators that show how effectively countries are preparing students to use mathematics in every aspect of their personal, civic and professional lives, as part of their constructive, engaged and reflective citizenship. To achieve this, PISA has developed a definition of mathematical literacy and an assessment framework that reflects the important components of this definition. The mathematics assessment items selected for inclusion in PISA 2015, based on this definition and framework, are intended to reflect a balance of relevant mathematical processes, mathematical content and contexts.
>
> …
>
> It is important that the construct of mathematical literacy, which is used in this report to denote the capacity of individuals to formulate, employ and interpret mathematics in a variety of contexts, not be perceived as synonymous with minimal, or low-level, knowledge and skills. Rather, it is intended to describe the capacities of individuals to reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena."
>
> (p.29–30, p.5 *PISA 2015 Draft Mathematics Framework*, 2013)

Mathematical literacy [...] describe[s] the capacities of individuals to reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena.

In 2015, 74 countries across the world took part in this international measure of learning, including the UK, the US and China.

What is *Progress Test in Maths*?

PTM is a series of assessments testing maths skills and knowledge for 5- to 14-year-olds. It can be used year-on-year and is suitable for both summative and formative purposes, to gain insight into students' strengths and weaknesses in maths, and to monitor progress made between the different test administrations. It has been developed in conjunction with the Mathematics Assessment Resource Service (MARS) at the University of Nottingham.

It supports the 'whole pupil view' by providing information on mathematics attainment, and helps identify areas where additional support or extension work are needed. The content has been developed to align with the frameworks of the different mathematics curricula in use in the UK.[2] These have been updated in recent years to reflect current research and global trends in mathematics education.[3]

For example, in England the purpose of study and aims of the mathematics curriculum are defined as follows:

Purpose of study

> "Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject."
>
> (p.3, *Mathematics programmes of study: key stages 1 and 2. National curriculum in England*, 2013)

[2] England, Wales, Scotland and Northern Ireland.

[3] See National curriculum in England: mathematics programmes of study and Education Scotland in the sources section of this document.

Aims

"The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems."

(p.3, *Mathematics programmes of study: key stages 1 and 2. National curriculum in England*, 2013)

Dimensions of mathematical competency

Both *PTM* and PISA have been developed from frameworks that identify the domain of mathematical competency as an area vital for success in life and in engaging with the world around us.

To measure these skills and knowledge, each item in both tests has different dimensions: a mathematical process category and a mathematical content category. PISA items also address the dimension of context. Test items in *PTM* have been developed with a view of being relevant to the real-life context of children in each age group, but this dimension is not separately assessed in the test, as focus is on mathematical understanding.

Both tests are based on the latest research and are measures of learning in mathematics, as well as the skills and knowledge expected from the age group being tested. Therefore, there is significant overlap in both the process and content categories of PISA and *PTM*.[4]

[4] For additional details on the process and content categories, please see *PTM Links to National Curricula* documents and report samples, and *PISA 2015 Draft Mathematics Framework*.

Process categories

Mathematical process categories describe working and thinking skills in mathematics beyond simple recall of facts and standard procedures. They relate to the use and application of mathematics.

PISA and *PTM* both address this in-depth learning and understanding, and assess several aspects important to mathematical competency. The language used to describe these processes varies, but the intentions are similar, as can be seen from the following table comparing PISA and *PTM* categories.

Mathematical process categories describe working and thinking skills in mathematics beyond simple recall of facts and standard procedures.

Mathematical process categories in PISA 2012 and 2015[5]	Mathematical process categories in *PTM*
• Formulating situations mathematically	• Mathematical reasoning
• Employing mathematical concepts, facts, procedures and reasoning	• Fluency in facts and procedures • Fluency in conceptual understanding
• Interpreting, applying and evaluating mathematical outcomes	• Mathematical reasoning • Problem solving

Information about skills in these process categories is presented in *PTM* reports on individual and group levels, in comparison to averages in the age group, to help identify areas for further development. This allows teachers to set individual and group targets in these categories and monitor attainment. With this data they are able to provide support for all students according to their individual needs.

Analysis of group scores (by Process category)

The table and bar chart below show the percentage of questions answered correctly by all students compared with those for the national average.

Curriculum content category	Number of questions	Group % correct	National % correct	Difference
Fluency in facts and procedures	13	69%	74%	-5%
Fluency in conceptual understanding	23	54%	66%	-12%
Problem solving	6	60%	38%	22%
Mathematical reasoning	12	47%	60%	-13%

Sample* PTM9 *Group Report for Teachers

The number of items in each process category varies depending on the *PTM* test level, with more focus on problem solving and reasoning in the higher levels.

[5] p.9–12, *PISA 2015 Draft Mathematics Framework*, 2013

Content categories

The content categories measure understanding and knowledge of mathematics. There are different ways of grouping mathematical content areas and different terms can be used to describe similar content. In basic education, mathematical terms are often replaced with terms from everyday language to make them more relatable to the experiences students have and contexts they are familiar with.

In PISA there are four content categories. These were identified in a study that looked at mathematical content taught in 11 countries to identify content that is forward-thinking, but also such that 15-year-old students were likely to have had an opportunity to learn it, and that is relevant to what is expected of them in further study and in the workplace. The previous mathematics curriculum for KS3 in England is one of the references cited in the PISA framework. The topics chosen for PISA "reflect commonalities found in the expectations set by a range of countries and educational jurisdictions." (p.19–20, p.52, p.17, *PISA 2015 Draft Mathematics Framework*, 2013) The new English curriculum - which has been used to develop *PTM* - is more demanding than the previous one. Therefore, students whose performance has been measured with *PTM* and appropriate steps taken based on the results, are expected to be well prepared for the demands of PISA.

The division of items to content categories is important for reporting purposes and for gaining a deeper understanding of each individual's learning against set targets. Where an item covers content from different categories the main content area is usually chosen when mapping items to categories.

The following table presents how the mathematical content categories in PISA relate to the six content categories used in *PTM*14 (number; algebra; ratio, proportion and rates of change; geometry and measures; probability; and statistics). Algebra falls under several, or all, of the PISA categories, depending on the test item. Some content topics may be addressed in more than one content category. (p.17, *PISA 2015 Draft Mathematics Framework*, 2013)

Mathematical process categories in PISA 2012 and 2015[6]	Mathematical content categories in *PTM*14
Change and relationships	Ratio, proportion and rates of change
Space and shape	Geometry and measures
Quantity	Number
Uncertainty and data	Probability, statistics

Information about skills in the different content categories is presented in *PTM* reports on individual and group level, in comparison to averages in the age group, to help identify areas for further development. This allows teachers to set individual and group targets for the different areas and monitor attainment. They are then also able to provide support for all students according to their individual needs.

[6] (p.17–19, *PISA 2015 Draft Mathematics Framework*, 2013).

Analysis of Curriculum content categories

Curriculum content category	Number of questions	Student % correct	National % correct	Student / national difference
Number	15	13%	37%	-24%
Algebra	15	6%	29%	-23%
Ratio, proportion and rates of change	7	29%	41%	-12%
Geometry and measures	21	22%	26%	-4%
Probability	4	0%	29%	-29%
Statistics	3	0%	59%	-59%

Sample PTM14 Individual Student Report for Teachers

The content categories differ for the age groups according to the skills expected from them, and the number of items in each category varies. This allows students to progress at age-appropriate pace and build a solid pathway to performing at age-expected level when they are 15 years old, which is when PISA is administered.

This allows students to progress at age-appropriate pace and build a solid pathway to performing at age-expected level ...

Assessment construction, outcomes and goals

While both assessments measure students' understanding in the field of mathematics, there are differences in the construction and purpose of the two tests, and these affect the extent to which it is meaningful to draw comparisons.

The differences do not mean that one test would be better than the other, or provide more 'correct' results. As demonstrated above, both assessments build on a shared understanding of what is meant by 'mathematical competency'. This encompasses dimensions from skills and knowledge to mathematical reasoning and problem solving. However, a comparison of any two different assessments must always take in to account what is being assessed, why it is being assessed, who is being assessed and how it is most appropriate to conduct the assessment.

The following table shows a comparison of these dimensions between *PTM* and PISA Mathematics.

PISA and PISA Mathematics Framework[7]	*Progress Test in Maths*
Purpose of the test: why measure? Assessments are always developed for a specific purpose. The reason for testing is what guides test construction and development.	
PISA aims to **evaluate education systems** worldwide by testing the skills and knowledge of 15-year-old students. A sample of students from each participating country represents the population, so that **comparisons between education systems** can be made. PISA is not designed to produce individual student scores.	*PTM* is developed to **assess a student's mathematical skills, knowledge and progress year-on-year**. The tests have been trialled and standardised on a sample of students, so that (individual) **comparisons to the age group** can be made.
Range of the test: what is being measured? To ensure test validity, that which is being measured must be clearly defined. Test items must contribute to providing a picture of the variable selected. A test may measure several variables, and to ensure reliability, there needs to be sufficient coverage of each variable.	
PISA measures **mathematical literacy** and covers three main domains: **mathematical processes, content knowledge** and **the context of the problem**. These are further split to different sub-domains. The mathematics section of PISA takes approximately 30 minutes to complete.	*PTM* measures **skills and knowledge in mathematics** and covers two main domains: **mathematical processes and content knowledge**. These are further split into different sub-domains. The test takes approximately 30 to 60 minutes to complete, depending on test level.
Target group: who is the test for? To develop a reliable test, it is necessary to define who the test is for and what kind of content is appropriate to the test takers, while enabling the measurement of the selected variable.	
PISA tests are taken by **a sample of 15-year-olds** in the participating countries. For the test's purpose, they represent the population of the country.	A separate, age-appropriate *PTM* test has been developed for **each age group from 5- to 14-year-olds**. The tests have been trialled and standardised on a comprehensive, **balanced sample of students in each age group** in the UK, so that comparisons to the age group can be made.
Method and format of the assessment: how to assess? There are different facets to the method of assessment and choosing the appropriate method depends on the purpose and variables of the test. For example, a test can be administered in a group or individually, it can be written or based on oral questions and observations or be a combination of the two, and it can be administered digitally or on paper.	
PISA is a **written test, administered to a group** of students at a time. For the first time in 2015, PISA was **delivered in digital format**, which was already available to some participants in 2012. A paper-based test was, however, still available for those countries that so chose. **All students do not take the same questions**, but each student is assigned a set of questions from a pool of test items to ensure system-wide coverage of the items.	*PTM* is also a **written test**, though for younger age groups the questions are read out loud and for older age groups there is a mental maths component, which is a voice recording. It is **administered in a group** and is available in **either paper or digital format** both of which have been standardised to ensure reliability across formats and sittings. **All students in each level take the same questions** to ensure each individual can demonstrate their learning in all areas that are being assessed.

[7] For sources, please see the end of this document.

The purpose of the two assessments is different, but there is significant overlap in content and methods. The strength of *PTM* is that it has been specifically developed and reliably standardised for each age group and helps identify individual needs, while PISA provides a snapshot of an education system's performance in a particular year with particular students.

The strength of PTM is that it has been specifically developed and reliably standardised for each age group and helps identify individual needs, while PISA provides a snapshot of an education system's performance in a particular year with particular students.

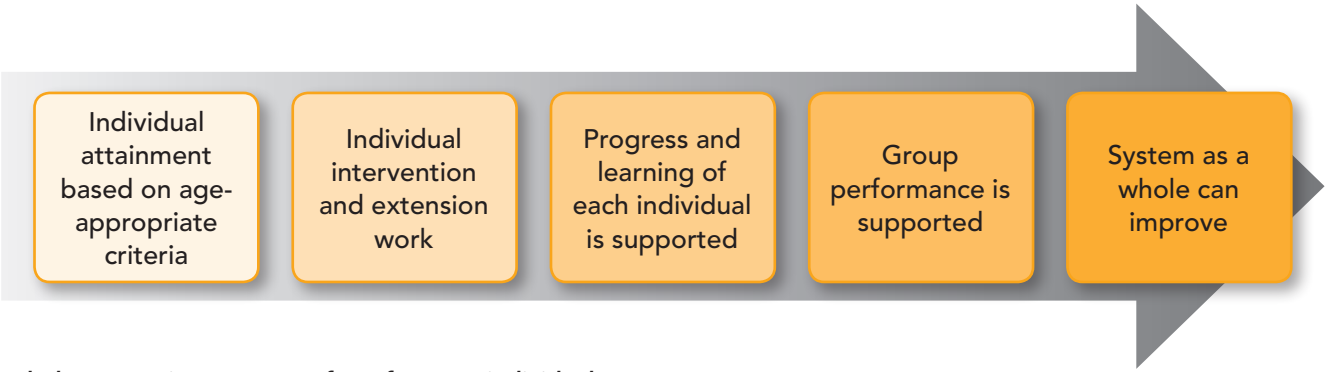
Can using *PTM* support success in PISA?

The two assessments measure similar concepts: mathematical thought processes and relevant mathematical content knowledge. As such, it can be assumed that on average a student who does well in one test, would perform well in the other test. Further, it can be inferred that where progress is made on *PTM* scores, similar progress would be reflected in PISA. It must again be highlighted that PISA is a measure of education systems, and *PTM* focuses on groups and individuals.

The PISA testing framework is readjusted for each triennial test run. While there were no significant changes to the Mathematics Framework between the 2012 and 2015 testing, the focus of PISA or the structure of the framework(s) may change from one test run to the next.

PTM provides a stable measure year-on-year for each individual and when a new edition is launched, a comparison to the previous edition is provided.

The advantage of using *PTM* is that as an assessment targeted for particular age groups, and carefully developed for children of that age, it is possible to identify an individual's strengths and weaknesses. Improvement of any education system must start at the level of supporting each individual to do their best, and *PTM* helps teachers with this.



Whole-system improvement from focus on individuals

PTM helps schools to support each individual student by providing accurate information on the mathematics progression of that individual. This, in turn, can be used to support that student in achieving their best in mathematics. When each individual is helped and supported to do their best, the overall mathematical attainment in a school – or country – will improve as well. PISA measures this improvement every three years, but only on 15-year-olds, and between very different systems.

Focus on the individuals and ensure each student is getting the individual interventions and extension work they need to perform their best.

A change in PISA ranking can sometimes mean simply that standards globally are being raised – and the only way to keep up is to focus on the individuals and ensure each student is getting the individual interventions and extension work they need to perform their best.

Sources

About PISA http://www.oecd.org/pisa/aboutpisa/ (accessed on: 04/01/2016)

Department for Education. *'Mathematics programmes of study: key stages 1 and 2. National curriculum in England.'* September 2013. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf (accessed on: 02/02/2016)

Department for Education. *'Mathematics programmes of study: key stage 3. National curriculum in England.'* September 2013. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/239058/SECONDARY_national_curriculum_-_Mathematics.pdf (accessed on: 02/02/2016)

Department for Education. *'National curriculum in England: mathematics programmes of study.'* September 2013. https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study (accessed on: 08/01/16)

Education Scotland. http://www.educationscotland.gov.uk/learningandteaching/curriculumareas/mathematics/ (accessed on: 08/01/16)

GL Assessment. *Progress Test in Maths 5-14.* 2015 and 2016.

GL Assessment. *Progress Test in Maths 5-14: 'Links to National Curricula.'* 2015 and 2016. http://www.gl-assessment.co.uk/*ptm*guidance (accessed on: 05/02/2016)

National Center for Education Statistics. *Participation in PISA by Year.* https://nces.ed.gov/surveys/pisa/countries.asp (accessed on: 09/02/2016)

PISA FAQ http://www.oecd.org/pisa/pisafaq/ (accessed on: 04/01/2016)

PISA. *PISA 2015 Draft Mathematics Framework.* March 2013. http://www.oecd.org/pisa/pisaproducts/Draft%20PISA%202015%20Mathematics%20Framework%20.pdf (accessed on: 04/01/2016)

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